Research on the Trading Strategy Model Based on Dynamic Programming Algorithm

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Abstract: This paper built three models to achieve daily purchase decisions and future property value estimates, the first is a dynamic planning purchase model, the second is the price grey prediction model, the third is the sale model. We take the forecast price of gold on the second day and the actual price on the first day as the starting point of analysis and use the ratio method to analyze and obtain the prediction coefficient. Then, the risk and yield in investing in gold and Bitcoin are initially taken as the starting point of analysis. It also adopts a mathematical method regarding the yield as a random variable and the risk as the coefficient representing the size of the yield fluctuation-variance. Weigh the return and risk of the portfolio by finding the Sharpe ratio is investigated. It is proven that our mathematical model provides the best strategy according to the Sharpe ratio calculations. Finally, we only change the value of trade cost in the case of controlling other condition variables unchanged. The control variable method is used to analyze the sensitivity of the strategy to trade costs. It is also continued to run our model when the trade costs increase or decrease by comparing the strategies given by the model under different trade costs under other conditions.

1. Introduction

In order to indicate the origin of problems, the following background is worth mentioning.

Gold is a tangible asset with good investment value. Gold is not easy to damage, can be stored for a long time, has low holding costs, and is highly sought after by investors. Investing in physical gold can be done by buying gold bars, coins, and jewelry. Gold jewelry surpasses the former two to become the best choice for ordinary people with its preservation and wearing functions.

Due to the rarity of gold, regardless of China and the West, people's understanding and feeling of gold make gold a frequent contact with supplies, decorations, jewelry consumer goods, and investment tools in people's lives.

Blockchain, short for BTC, is based on decentralized, peer-to-peer network and consensus initiative, open-source, and blockchain as the underlying technology. Satoshi Nakamoto published Bitcoin on November 1, 2008 bitcoin white paper "Bitcoin: A peer-to-peer electronic cash System proposed that On January 3, 2009, Trends block was born, and Bitcoin officially entered the historical stage.

Below, the article builds a mathematical model of how $1,000 as a principal will be purchased for gold and bitcoin to maximize profits and achieve the goal of estimating the property's value on a specified date.

2. Decision making and value model

The requirement to give the best investment strategy and estimate the final value as of September 10, 2021, we can divide into two problems to solve.

First, we need to address the day's decisions. In the decision-making process, we should understand that the decision of the first day depends on the return of the second day, so we built a price prediction model for gold and Bitcoin based on the gray prediction model. For the problem of how to buy gold and Bitcoin, we use dynamic programming algorithms to solve them.

First, let us discuss the amount of gold and bitcoin purchased. Without considering the commissions generated by trading, assuming that gold is bought in y DOLLARs, the amount of money we use to buy Bitcoin is (1000-y) DOLLARs. Let g(y) be the proceeds for the purchase of gold and f(1000-y) the proceeds for the purchase of Bitcoin. Well, for the total return W invested with the $1,000 model, it is clear that there is the following relationship:

$$W = g(y) + f(1000 - y) \tag{1}$$

According to the principle of dynamic programming, we can get the following relation:

$$W = \max_{0 \le y \le 1000} \{g(y) + f(1000 - y)\} \tag{2}$$

The above discussion is only carried out based on not considering transaction costs (i.e., commissions), and now we add commissions to the discussion. According to the description in the text, it is easy to know: the commission with y dollars is the y*a% dollar. Similarly, the transaction cost of buying Bitcoin is (1000-y)*ω% USD. Then, taking into account the commission cost, the total income W for the investment of $1000 satisfies the following relationship:

$$W = g(y) + f(1000 - y) - [y \times \alpha\% + (1000 - y) \times \omega\%] \tag{3}$$

According to the principle of dynamic programming, we can get the following relation:

$$W = \max_{0 \le y \le 1000} \{g(y) + f(1000 - y) - [y \times \alpha\% + (1000 - y) \times \omega\%]\} \tag{4}$$

After obtaining the following relational equation, we can use the principle of the exhaustive method to solve the maximum value. In order to avoid the heavy process, we program it in C. The profit generated by the purchase of gold and bitcoin depends on the actual price of gold and bitcoin the next day. According to the description in the title, we cannot know the actual price of the next day. Therefore, according to the basic principles of the gray prediction model, we use MATLAB software to program. Based on the predicted prices of gold and Bitcoin, we calculated the forecast profit of one unit of gold and Bitcoin on the next day and used the dynamic programming model and its procedural solution to decide how to buy gold and Bitcoin on the day.

As for whether to sell our holdings of gold and bitcoin, let us first discuss if gold can be sold at any time. We might as well define a coefficient that reflects the suitability to sell gold and Bitcoin, which we call the prediction factor 2. For the prediction coefficient h, we can artificially divide the forecast for gold and Bitcoin the next day from the actual values of gold and Bitcoin on the first day. Thus, we can get the following relational formula:

$$\lambda = \frac{\psi}{\Omega} \tag{5}$$

Among them, ψ it represents the predicted value of gold and Bitcoin for the next day, and Ω represents the actual price of gold and Bitcoin. λ is the prediction factor.

After obtaining the prediction coefficient λ , we λ determine the prediction coefficient, if the λ > 1, it means that gold and Bitcoin still have room to continue to grow, we do not sell it, if λ < l, it means that gold and Bitcoin have been We cannot continue to add value, we need to sell it. The specific flow chart is as follows in Figure 1:

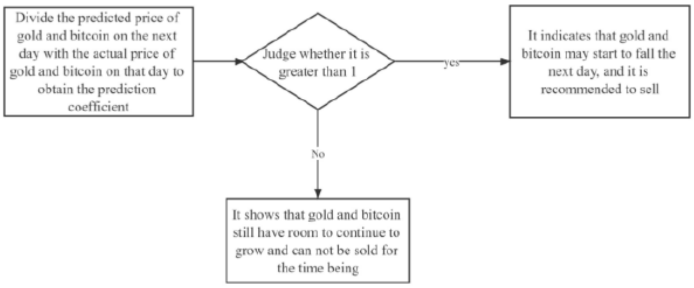

Figure 1. The specific flow chart

Below, we discuss how to sell gold to reach the goal of making a profit by considering the gold trading open day.

For the situation of considering the open day of gold trading, there are only three cases:

(1) On the day of the Gold Trading Open, it coincides with the conditions we plan to sell for gold.

(2) On the day of the Gold Trading Open, it is eligible for our planned non-sale of gold

(3) The day we plan to sell gold is not an open day for gold trading.

Next, let us start discussing the above three scenarios:

We only need to sell the gold as planned for cases (1) and (2). For case (3), we need to do something else.

For case (3), we can solve it in the following way, since the price of gold changes with the number of days. Therefore, the day before the open trading day, we can get the next day's gold forecast price according to the forecast model, so that we can get the forecast profit of gold. After getting the forecast profit of gold, we determine the forecast profit of gold, and if the predicted profit cost of gold>0, we can sell it. If the projected cost of gold <0, then we take the option of not selling it. The specific flow chart is as follows as Figure 2:

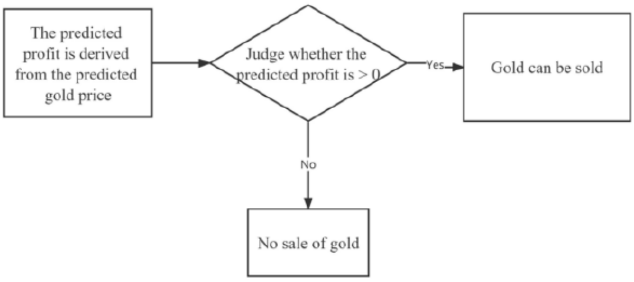

Figure 2. Flow chart of whether gold is sold

Below, we can calculate the earnings for each day based on the process above to estimate the original $1,000 property value as of September 10, 2021. If we want to figure out how much the initial $1,000 is worth as of September 10, 2021, we only need to cycle the number of days in days until 2021. At the end of the cycle on September 10, we can develop the initial $1,000 value.

After calculating the above process, we finally concluded that as of September 10, 2021, the value of the initial $1000 investment was $5798.874.

3. Return and risk model based on variance

We think of the return of security as a random variable and the return as the mathematical expectation of this random variable. The risk as to the coefficient of the magnitude of the fluctuation of this random variable is the variance. Variance is a penalty for an asset's return on deviation from the expected rate of return, and it indicates a reliable coefficient of asset performance. First, the variance of a single asset:

$$E(X) = \sum_{i=1}^{n} X_i P_i \tag{6}$$

$$\sigma^2(X) = \sum_{i=1}^{n} [X_i - E(X)]^2 P_i \tag{7}$$

Where X_i, P_i are the possible rate of return and its corresponding probability. Second, the variance of the portfolio of securities:

$$S_P = \sigma_P^2(X) = \sum_{i=1}^{n} \sum_{i=1}^{n} w_i w_j \text{cov}(X_i, X_j) \tag{8}$$

Where w_i, w_j the weights for each asset are assigned, the $\text{cov}(X_i, X_j)$ covariance of the asset.

It is easy to know that standard deviation, $\sigma(X)$ as a measure of risk, has the following properties:

(1) The standard deviation exists when the second-order moment of stochastic risk X is present.

(2) For any ß > 0, $\sigma(\beta X) = \beta\sigma(X)$;

It is indicated that the size of the risk changes in proportion to the scale of investment, where when the asset size drops to zero, it corresponds to the zero-risk situation.

(3) $\sigma(\sum_{i=1}^{n} w_i X_i) \leq \sum_{i=1}^{n} w_i \sigma(X_i)$ where w_i is a positive actual number.

It is shown that diversification can play a role in reducing risk. The advantage of the variance or standard deviation method is that the measure of risk is bidirectional, which facilitates the application of the statistical technique of normal distribution. However, there are also some deficiencies:

The variance calculation is stuck in the analysis of historical data, but the asset's future performance will change to a certain extent, so there will be a specific error in predicting future investment.

Although there are shortcomings, through the variance, we can more accurately derive the optimal solution based on the solution of the quadratic planning as a guide for investment planning. In the financial securities market, the Sharpe ratio can measure the return and risk of the portfolio at the same time and is the most commonly used indicator in the performance evaluation of the fund, which can provide investors with an essential guiding role in the portfolio.

By consulting the 1iterature, we found that the main methods used to calculate sharp ratio functions are divided into indirect and direct methods. Since the Sharpe ratio is the ratio of excess returns to portfolio volatility, if the indirect method is used, the problem is converted to solving the mean and standard deviation of the yield series separately and then calculating the Sharpe ratio function. If the direct law problem is used, the ratio of the mean and variance of the yield series is directly calculated as a whole to calculate the Sharpe ratio function. Recently, there are many ways to calculate the mean and variance, the simplest of which is the moment estimation method, which uses the sample first-order moment to estimate the population expectation, and the second-order sample center distance to estimate the variance, that is, the mean and variance of the population are estimated by using the historical yield series data. The mean function and the variance function need to be modeled when estimating the Sharpe ratio function, and the current prediction model for volatility can be divided into historical information and implied volatility methods. GARCH models in the historical information method can use historical data in financial markets to find hidden patterns between time series. The GARCH model is based on the ARCH model after further research. It is simpler than the ARCH model, and it adds the lag period of the error term conditional variance. The impact factor is extended to the mean squared error and the pre-standard value of the conditional variance, which can better solve the problem of volatility clustering than the ARCH model, which can better solve the problem of volatility clustering and effectively eliminate excessive peaks in asset return.

Furthermore, the GARCH model can be built to solve the income sequence. The ARCH effect of correlation effectively measures the risk level of gold and Bitcoin yields under the distribution of volatility agglomeration, thus better describing financial time series. Recently, the GARCH model has been widely used to study the volatility of economic and financial time series. The result of Sharpe ratios is not affected by the GARCH process and is only related to the coefficients of the ARMA model part (the mean equation), so after the optimal model of the yield series ARMA (p, q) - GARCH (r, s), only the coefficients of the mean function part need to be used to calculate the Sharpe ratio. The method consists of nine steps.

Step 1: Set the closing price sequence of gold and Bitcoin to p_1, P_2, P_3, ..., P_n, calculate the corresponding logarithmic return as:

$$r_t = \ln\frac{p_t}{p_{t-1}}, t = 1, 2, \cdots, n \tag{9}$$

Step 2: Build a model under conditions where the logarithmic yield sequence satisfies the stationarity ARMA (p, q):

$$r_t = \sum_{i=1}^{p} \varphi_i r_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (10)$$

Where ε_t for independent Gaussian white noise, $\varepsilon_t \sim N(0, \sigma^2), E(x_t \epsilon_t) = 0$.

Step 3: Under the known pre-t period information, calculate the yield of t+1 period, which is:

$$r_t = \sum_{i=1}^{p} \varphi_i r_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (11)$$

Step 4: Calculate the variance of the t+1 period, i.e., the autocovariance γ_0.

$$r_t - \sum_{i=1}^{p} \varphi_i r_{i-t} = \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (12)$$

$$\left(1 - \sum_{l=1}^{p} \varphi_i B^i\right) r_t = \left(1 + \sum_{t=1}^{q} \theta_i B^i\right) \varepsilon_t \quad (13)$$

Where $r_t = \sum_{j=1}^{\infty} G_j B^j \epsilon_t$, the two ends of the equation are multiplied by the same time r_t and then taken as expected, it can be obtained:

$$\gamma_0 = \sigma^2 \sum_{j=0}^{\infty} G_j^2 \quad (14)$$

Bring $r_t = \sum_{j=0}^{\infty} G_j B^j \epsilon_t$ in formula (12), get:

$$\left(1 - \sum_{i=1}^{p} \varphi_i B^t\right) \sum_{j=0}^{\infty} G_j B^j \varepsilon_t = \left(1 + \sum_{i=1}^{q} \theta_i B^l\right) \varepsilon_t \quad (15)$$

The specific values that can be determined according to the pending coefficient method G_j are:

$$G_j = \begin{cases} 1, i = 0 \\ \theta_j + \sum_{i=1}^{p} \varphi_i + G_{j-t}, j = 1,2,3,\cdots,q \\ \sum_{i=1}^{p} \varphi_i G_{j-i}, j > q, j - i \geq 0 \end{cases} \quad (16)$$

Step 5: Let the prediction origin be t, F_t, which is a collection of information that can be obtained at t-time, then t+1 is known at t-time and before t-time. The period of the one-step ahead is predicted as:

$$r_{t+1}^{\wedge} = E(r_{t+1} \mid F_t) = E\left(\sum_{i=1}^{p} \varphi_i r_{i+1-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t+1-i} + \varepsilon_{t+1} \mid F_t\right) = \sum_{i=1}^{p} \varphi_i r_{i+1-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t+1-i} \quad (17)$$

Further, equation (10) can be expressed as:

$$r_{t+1} = r_{t+1}^{\wedge} + \varepsilon_{t+1} \quad (18)$$

Step 6: Assuming that $r_{t+1}^{\wedge} \sim N(0, \sigma_E^2)$, ε_{t+1} the and $r_{t+1}^{\wedge}$ are independent, the equivalence relationship between the variances of the three is:

$$\sigma_E^2 = \gamma_0 - \sigma^2 \quad (19)$$

Step 7: Develop an investment strategy. The strategy of maximizing the expectation of a single period of excess return at t-time, assuming a risk-free rate of return and zero-cost trading rate, can be expressed as follows: if the advance prediction of the ARMA-GARCH model $r_{t+1}^{\wedge} > 0$ is to buy the index at the closing price. If $r_{t+1}^{\wedge} < 0$ so, to sell the index at the closing price.

4. Income analysis based on model comparison

For the determination of the sensitivity of the strategy to transaction costs and how the transaction costs affect the strategy and results, we chose the control variable method, in the case of controlling other conditions, only change the variable of transaction cost, run the changed model, and test the change of transaction decision results under different transaction costs. Knowing the transaction cost of the original topic, the transaction cost of gold is 1%, the transaction cost of Bitcoin is 2%, we distribute assuming two other sets of data, the first group we set the transaction cost of gold as 0.5%, and the transaction cost of Bitcoin as 1.5%. In the second group, we set the transaction cost of gold at 2%, the transaction cost of Bitcoin at 3%, and the program was run again to get:

Take April 5, 2017, as an example, the price of gold on that day was $1245.8 per Troy ounce, and the price of Bitcoin was $1133.079314 per Bitcoin. We predicted the outcome for gold at $1255.2278 per trojan; Bitcoin at $1069.2022 per bitcoin when gold transaction costs 1%. When the bitcoin transaction cost is 2%, according to the mode1, we should buy 2.09 units of Bitcoin, not buy gold, and when the transaction cost is reduced to 0.5 of the first group % and 1.5%, the model gives the decision not to buy gold, to buy 2.11 units of Bitcoin. When the transaction cost rose to 2% and 3 in the second group at the time of the program, our decision was not to buy gold according to the procedure but to buy 2.09 units of Bitcoin.

For example, on April 26, 2017, according to the original transaction cost, we decided to buy 2.17 troy ounces of gold, not to buy Bitcoin, and according to the assumptions of the first group, our decision was changed to buy 2.19 trojans of gold without buying Bitcoin; according to the assumptions of the second set, we decided to buy 2.17 troy ounces of gold instead. Bitcoin purchases are not made.

After several tests, we found that when transaction costs decrease when we need to buy more gold and bitcoin, the decision will allow us to buy more units, and when the transaction cost rises, our decisions will remain the same. Moreover, when the transaction cost is reduced, the changed decision will make us more profitable, we can get more funds to invest, and when the transaction cost increases, the decision almost does not change, which also shows the stability of our model, the strategy will not be affected too much by the increase in transaction costs. When the transaction cost is reduced, the decision will also be adjusted by purchasing more to improve the income, which also reflects the sensitivity of the forecasting strategy given by our model to the transaction cost.

5. Assessment and advancement of the model

When the model is set up, we need to make a reasonable assessment of the advantages and disadvantages and put forward a sensible recommendation for moving forward the model.

5.1. The advantages of the model

(1) Dynamic programming in operations research is used when building models to seek decision optimization by building equations. It is known that this problem belongs to the multi-stage decision-making problem. In the multi-stage decision-making problem, the decisions taken at each stage are generally time-related, the decision- making depends on the current state, and then causes the transfer of the state, a decision sequence is generated in the changing state, using the dynamic programming method so that our model can propose the best decision.

(2) When testing the sensitivity of the model to transaction costs, it takes the method of controlling variables, which only change the variable of transaction costs. Then, it is carried out program operations and model tests, which can more intuitively reflect the impact of changes in transaction costs on strategies and results, and is also relatively simple and convenient, there is no need to re-establish a new model to increase workload, and the results obtained are more obviously different in comparison. Fully reflect the sensitivity of the model to transaction costs.

(3) This article defines the yield and risk in the trade as the variance of a random variable and the random variable, respectively. The variance is the degree of punishment for the deviation of the return on the asset from the expected rate of return, which indicates the reliability coefficient of the asset's performance. The advantage of the variance method is that the measure of risk is bidirectional, which facilitates the application of the statistical technique of normal distribution.

5.2. The drawback of the model

(1) Due to the time relationship, our consideration of this mathematical model is not very careful, and the problem of inconsistency between the open day of the gold trading market and the date of the best trading decision is not solved very well. However, it is sold in the case of gold to obtain profits, without considering the problem of whether it is the most profitable.

(2) The calculation of variance stays on the analysis of historical data, but the future performance of assets will produce specific changes, so there will be a particular error performance in predicting future investment.

6. Summary

We built three models to achieve daily purchase decisions and future property value estimates. The first is a dynamic planning purchase model, and we aim at how to buy gold and Bitcoin to maximize profits, using dynamic programming algorithms to solve them, and finally, through C Language programs to get the best decisions every day. The second is the price grey prediction model. When making a profit, we should note that the first day's buying strategy essentially determines the profit and loss of the second day. Therefore, to know how to buy to get the most profit, we should know the predicted prices of gold and Bitcoin the next day. When we get the predicted price, we get the forecast profit and then adopt the best buying strategy. The third is the sale model, and for gold and bitcoin that has been able to make a profit, at what point in time will gold and bitcoin be sold is also a question. For this problem, we take the forecast price of gold on the second day and the actual price on the first day as the starting point of analysis and use the ratio method to analyze and obtain the prediction coefficient λ. After obtaining the prediction coefficient λ, we determine the prediction coefficient λ, if the $\lambda > 1$, it means that gold and Bitcoin still have room to continue to grow, we do not sell it, if $\lambda < 1$, it means that gold and Bitcoin have been We cannot continue to add value, we need to sell it. To predict the value of the property on a future day, we cycle in days until the end of the cycle to the target date, and then we can get the property value estimate of the target date.

We take the risk and yield in investing in gold and Bitcoin as the starting point of analysis adopt a mathematical method, regarding the yield as a random variable. The risk as the coefficient represents the yield fluctuation's size- variance, weighs the portfolio's return and risk by finding the Sharpe ratio, and proves that our mathematical model provides the best strategy according to the Sharpe ratio calculation. We only change the value of trade cost ah% in the case of controlling other condition variables unchanged, use the control variable method to analyze the sensitivity of the strategy to trade costs, and continue to run our model when the trade costs increase or decrease by comparing the strategies given by the model under different trade costs under other conditions. The final benefit result reflects the sensitivity of our strategy to transaction costs and the impact of transaction costs on the strategy and outcome.

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